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# A Slavnov-Taylor identity and equality of damping rates for static transverse and longitudinal gluons in hot QCD

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**Abstract**

A Slavnov-Taylor identity is derived for the gluon polarization tensor in hot QCD. We evaluate its implications for damping of gluonic modes in the plasma. Applying the identity to next to the leading order in hard-thermal-loop resummed perturbation theory, we derive the expected equality of damping rates for static transverse and longitudinal (soft) gluons. This is of interest also in view of deviating recent reports of $\gamma_t(p=0) \neq \gamma_l(p=0)$ based on a direct calculation of $\gamma_l(p=0)$.

*Key words:* hot QCD, Slavnov-Taylor identity, damping rate, gluon
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## 1 Introduction

Much interest is devoted to the physics of a the deconfinement phase of hadronic matter (quark-gluon plasma, QGP), with both strong experimental and theoretical research going on. A good theoretical understanding of this new phase of matter is urgently required, in particular, in order to identify worthwhile observables of this new phase of matter. Insight can be gained in the framework of 'Hot QCD' supplemented with a perturbative approach and important progress has been made during the last decade [1]. In particular thermal effects are known to alter soft modes in an important way, changing dispersion relations, and allowing for Landau damping as well as for new modes of propagation for both fermionic (quarks) and bosonic (gluons) quasiparticles [2]. The collective dynamics involved can be consistently described within Hard-Thermal-Loop (HTL) resummed perturbation theory [3,4].

In this letter we are interested in damping of collective gluonic excitations as one of the most important characteristics of the plasma dynamics. While on-shell damping is known to be absent to HTL order $g^2T^2$ a non zero damping rate is expected to arise at order $g(g^2T^2)$. In early work by Braaten and Pisarski [5] a finite, gauge independent result for the damping rate of static transverse gluons $\gamma_t(\omega = m_g, p = 0)$ has been obtained, as one of the important applications of the HTL-resummation scheme. Note that gauge invariance also follows from a general theorem [6]. Also, in the static regime, $\gamma_l(m_g, p = 0) = \gamma_t(m_g, p = 0)$ has been mentioned in [5], as is expected since longitudinal (l) and transverse (t) degrees of freedom can not be distinguished at zero spatial momentum. However, a complete understanding of the damping behavior in the plasma is not yet available. In particular, for non-zero values of the spatial momentum singular results are obtained for transverse and also for longitudinal gluons [7], which originate from the absence of a chromo-magnetic mass in the HTL resummed gluon propagator. Moreover, also the analyticity of the damping rates around zero momentum has been questioned recently [8]: Expansion for small $p$ of the imaginary part of the gluon polarization tensor has been reported to lead to $\gamma_t(m_g, 0) \neq \gamma_l(m_g, 0)$ and even to a quadratic divergence in $\gamma_l(m_g, p = 0)$.

In this letter we derive a Slavnov-Taylor (ST) identity for the gluon polarization tensor in Coulomb gauge. After discussing some general consequences of it, we apply it to the next to leading order contribution to the gluon polarization tensor ${}^*\Pi^{\mu\nu}$, which might be helpful in order to further clarify the physics of damping for gluonic modes in the plasma. From it we easily obtain well known transversality properties as well as new constraints in particular concerning the imaginary part of ${}^*\Pi^{\mu\nu}$ related to damping. The expected equality $\gamma_l(m_g, 0) = \gamma_t(m_g, 0)$ can also be derived. We report on these findings in the next two sections and further implications are discussed in the conclusions

section. For convenience, the derivation of the ST identity is deferred to the Appendix.

## 2 Slavnov-Taylor identity and its consequences

Within the imaginary-time formalism of hot QCD, ST identities are derived from the BRST invariance of the Lagrangian in the same way as in vacuum theory. For a discussion of dynamical quantities continuation to Minkowski space should be done in the usual way. We briefly summarize the procedure in the Appendix.

Most important to the dynamics of gluonic fields is the identity involving the gluon polarization tensor $\Pi^{\mu\nu}$. In Coulomb gauge it reads:

$$P_\nu \Pi^{\mu\nu}(P) = - \left[ \delta^\mu{}_\nu P^2 - P^\mu P_\nu + \Pi^\mu{}_\nu(P) \right] \Pi_g^\nu(P) \,, \tag{1}$$

with $\Pi_g^\nu(P)$ related to the Coulomb-ghost self-energy part $\Pi_g(P)$ through $p^i \Pi_g^i(P) = \Pi_g(P)$.[4] The derivation of the result (1) is explained in the Appendix. Note that it is an exact identity, valid for arbitrary momenta $P$ and to all order in perturbation theory.

¿From Eq. (1), other identities are immediately obtained that will be useful for the discussion below. First contracting also the remaining Lorentz index we obtain

$$\begin{aligned} P_\mu \Pi^{\mu\nu}(P) P_\nu &= - P_\mu \Pi^\mu{}_\nu(P) \Pi_g^\nu(P) \\ &= \left[ P^2 g_{\mu\nu} - P_\mu P_\nu \right] \Pi_g^\mu \Pi_g^\nu + \Pi_g^\mu \Pi_{\mu\nu} \Pi_g^\nu \,. \end{aligned} \tag{2}$$

Concerning the imaginary part of $\Pi^{\mu\nu}(P)$, since[5] $Im \Pi_g^\nu(P) = 0$, we obtain

$$P^\nu Im \Pi_{\mu\nu}(P) = - \Pi_g^\nu(P) Im \Pi_{\mu\nu}(P) \,. \tag{3}$$

$$P^\mu Im \Pi_{\mu\nu}(P) P^\nu = \Pi_g^\mu \Pi_g^\nu Im \Pi_{\mu\nu} \,. \tag{4}$$

[4] Unless otherwise stated, throughout in this letter, Greek letters, $\mu$, $\nu$, ..., take $0, 1, 2, 3$ while Ratin letters, $i$, $j$, ..., take $1, 2, 3$.

[5] This can readily be seen in real-time formalism [4]. The building blocks of perturbation theory, i.e., the propagators and vertices, and then also the self-energy part, are $2 \times 2$ matrices in "thermal space." $Im \Pi_g^\nu(P)$ here is proportional to the $(1, 2)$-component of $(\Pi_g^\nu(P))_{ij}$ $(i, j = 1, 2)$. Since, in Coulomb gauge, the ghost propagator-matrix $(\Delta_g(P))_{ij}$ $(i, j = 1, 2)$ is diagonal in thermal space, $(\Pi_g^\nu(P))_{ij}$ is also diagonal and $(\Pi_g^\nu(P))_{12} = 0$.

Note again that, as Eq. (1), also Eqs. (2) - (4), are general, valid for arbitrary momenta and to all orders in perturbation theory.

Eqs. (1) - (4) contain important informations. At leading and next to leading order several of them have been established through direct calculation. Consider the soft momentum region, $P^\mu = (\omega, \mathbf{p}) \sim gT$; from Eqs. (1) and (2) we readily rederive two well known results from power counting arguments:

(1) At lowest nontrivial order of effective (HTL-resummed) perturbation theory [3,4] $\Pi^{\mu\nu}(P)$ $(= \delta\Pi^{\mu\nu}(P)) = O(g^2T^2)$ so the left-hand side (LHS) of Eq. (1) is of $O(g^3T^3)$. On the other hand $\Pi_g^\nu(P) = O(g^2T)$ (no HTL-contribution in amplitudes with external ghosts [3,4]) and the right-hand side (RHS) of Eq. (1) is of $O(g^4T^3)$. Therefore, to the HTL accuracy, $P_\nu\Pi^{\mu\nu}(P) \simeq P_\nu\delta\Pi^{\mu\nu}(P) = 0$.

(2) Similarly, at leading order, the LHS of Eq. (2) is of $O(g^4T^4)$. In the second line of Eq. (2), $\Pi_g^\mu(P) = O(g^2T)$, $\Pi^{\mu\nu}(P) = \delta\Pi^{\mu\nu}(P) = O(g^2T^2)$, $\Pi_g(P) = O(g^3T^2)$. Therefore, RHS/LHS $= O(g^2)$, and

$$P_\nu\Pi^{\nu\mu}(P)P_\mu = 0 \tag{5}$$

holds to relative order $g^2$. A more precise statement is derived below, cf. Eq. (8).

We now proceed exploiting the results Eqs. (1) - (4) in more detail. We focus on the soft momentum region and the next to leading order contribution to $\Pi^{\mu\nu}(P)$ from which the leading damping behavior will be deduced. Following standard notations, we write $\Pi^{\mu\nu}(P)$ to this order $\delta\Pi^{\nu\mu}(P) + {}^*\Pi^{\nu\mu}(P)$ with $\delta\Pi$ the HTL-contribution and ${}^*\Pi$ the relative order $g$ correction. The diagrams that yield leading contributions to the imaginary part of ${}^*\Pi$ are shown in Fig. 1 below.

In Eqs. (1) - (4) we need an explicit expression for the ghost contribution. It is sufficient to calculate $\Pi_g^\mu(P)$ to lowest nontrivial order, which we carry out in Appendix (cf. Eq. (32)):

$$\Pi_g^\mu(P) \simeq -\delta^{\mu i}\frac{g^2NT}{16}\hat{p}^i\,, \tag{6}$$

where $\hat{p}^i = p^i/p$. It is worth mentioning in passing that $\Pi_g^\mu(\omega, p = 0) = 0$. This is because, for $\mathbf{p} = 0$, the summand/integrand of Eq. (30) in Appendix, which is valid for arbitrary (Euclidean) four-momentum $P_E$, is odd in the summation/integration variables $K_E$.

First we use Eq. (6) in Eq. (1): To next to leading order under consideration we can write $\delta\Pi^\mu{}_\nu$ for $\Pi^\mu{}_\nu$ on the RHS of Eq. (1) and using $\delta\Pi^\mu{}_\nu P^\nu = 0$ [3,4] as well as (5) we find

$$P_\nu {}^*\Pi^{\nu\mu}(P) = \frac{g^2NT}{16}\frac{\omega}{p}{}^*\Delta_l^{-1}(P)\left(1,\, \frac{\omega}{p}\hat{\mathbf{p}}\right)^\mu \tag{7}$$

with the inverse plasmon propagator [3,4],

$${}^*\Delta_l^{-1}(P) = p^2 - \delta\Pi_{00}(P) = p^2 + 3m_g^2\left[1 - \frac{\omega}{2p}\ln\left(\frac{\omega+p}{\omega-p}\right)\right],$$

where $m_g^2 = N(gT)^2/9$. Note that the close connection with the inverse propagator is of course apparent in Eq. (1). Also Eq. (7) is valid for any value of $\lambda$, the Coulomb gauge parameter, Eq. (16) (in Appendix). We have confirmed Eq. (7) through direct calculation of the LHS.

Incidentally, from Eq. (7), $P_\mu {}^*\Pi^{\mu i}(P)e_i^{(r)}(\hat{\mathbf{p}}) = 0$, $(r = 1, 2)$ for any three-vector $e_i^{(r)}(\mathbf{p})$ perpendicular to $\hat{\mathbf{p}}$ and arbitrary (soft) $P^\mu$.

Using Eq. (6) in Eq. (2) we find

$$P_\mu {}^*\Pi^{\mu\nu}(P)P_\nu = -\left(\frac{g^2NT\omega}{16}\right)^2\left[1 - \frac{1}{\omega^2}\hat{p}^i\delta\Pi^{ij}(P)\hat{p}^j + O(g)\right], \tag{8}$$

which is of $O(g^6T^4)$ when $P^\mu$ is soft, thus specifying somewhat the result under point (2) above.

Finally, making progress towards the discussion of damping in the next subsection, we note that, from Eq. (7), the imaginary part of $P_\mu {}^*\Pi^{\mu\nu}(P)$ is related to the Landau damping contribution in the HTL-propagator. We have:

$$Im\left[P_\mu {}^*\Pi^{\mu\nu}(P)\right] = \frac{3\pi}{32}g^2NTm_g^2\left(\frac{\omega}{p}\right)^2\theta(p^2-\omega^2), \tag{9}$$

and therefore for time-like momenta,

$$Im\left[P_\mu {}^*\Pi^{\mu\nu}(P)\right] = 0 \qquad (P^2 > 0). \tag{10}$$

## 3 Damping in a gluonic Medium

Soft gluon damping rates relate to the imaginary part of the gluon polarization tensor $\Pi^{\mu\nu}$, leading damping arises from the relative order g correction ${}^*\Pi^{\mu\nu}$. The relevant diagrams are shown in Fig. 1 where however the Coulomb-ghost loop does not contribute to $Im\Pi^{\mu\nu}$. HTL resummed propagators and vertices

have to be used since the dominant contribution arises from configurations with soft loop momentum. For the following presentation we restrict to strict Coulomb gauge ($\lambda = 0$ in Eq. (17) in Appendix).

For gluon damping only the transverse and longitudinal components of ${}^*\Pi^{\mu\nu}$

$$
\begin{aligned}
{}^*\Pi_t(P) &= \frac{1}{2}\delta_\perp{}^{ij}(\hat{\mathbf{p}})^*\Pi^{ij}(P)\,, \quad \delta_\perp^{ij}(\hat{\mathbf{p}}) = \delta^{ij} - \hat{p}^i\hat{p}^j\,, \\
{}^*\Pi_l(P) &= -{}^*\Pi^{00}(P)\,.
\end{aligned} \tag{11}
$$

will be relevant. In the vicinity of $\omega = \omega_{t/l}(p)$, the mass-shell for quasiparticles to HTL-accuracy, the respective inverse propagator component can be written:

$$
\begin{aligned}
G_{t/l}^{-1} &\simeq \mp Z_{t/l}^{-1}(p)\,(\omega - \omega_{t/l}) + i Im^*\Pi_{t/l}(\omega = \omega_{t/l}, p)\,, \\
Z_t^{-1}(p) &= \frac{3m_g^2\omega_t^2 - (\omega_t^2 - p^2)^2}{\omega_t(\omega_t^2 - p^2)} \stackrel{p\to 0}{\to} 2m_g\,, \\
Z_l^{-1}(p) &= 3p^2\left[\frac{m_g^2}{\omega_l(\omega_l^2 - p^2)} - \frac{1}{3\omega_l}\right] \stackrel{p\to 0}{\to} \frac{2p^2}{m_g}\,.
\end{aligned}
$$

¿From this, the damping rates for longitudinal and transverse gluonic modes are obtained as:

$$
\begin{aligned}
\gamma_t(\omega_t(p), p) &= -Z_t(p)\, Im\, {}^*\Pi_t(\omega_t(p), p) \quad \stackrel{p\to 0}{\to} \quad -\frac{1}{2m_g}\, Im\, {}^*\Pi_t(m_g, 0)\,, \\
\gamma_l(\omega_l(p), p) &= +Z_l(p)\, Im\, {}^*\Pi_l(\omega_l(p), p) \quad \stackrel{p\to 0}{\to} \quad \frac{m_g}{2p^2}\, Im\, {}^*\Pi_l(m_g, 0)\,,
\end{aligned} \tag{12}
$$

with $m_g = \omega_{t/l}(p = 0)$ the plasma frequency.

The direct calculation of $\gamma_{l/t}$, for general p, has not yet been accomplished, however a leading logarithmic singularity has been extracted for non-zero momentum in both $\gamma_t$ and $\gamma_l$ [7]. At zero momentum the calculation of $\gamma_t$ is well documented in the literature but note that calculating $\gamma_l$ requires to expand ${}^*\Pi_l(P)$ to order $O(p^2)$ - a formidable and subtle task. A recent controversy [8] suggests that approximations should be handled with care and observing general constraints.

We are now in a position to show that our results so far leads to $\gamma_l(m_g, 0) = \gamma_t(m_g, 0)$ and thus a consistent calculation necessarily provides it. On the gluon mass-shell, $P^2 = \omega^2 - p^2 > 0$, we obtain from Eq. (5) and Eq. (10)

$$
Im^*\Pi^{00}(P) = -Im^*\Pi_l(P) = \left(\frac{p}{\omega}\right)^2 \hat{p}^i\hat{p}^j Im^*\Pi^{ij}(\omega, \mathbf{p})\,. \tag{13}
$$

Specializing to $p = 0$ and using the fact ${}^*\Pi^{ij}(\omega, \mathbf{p} = \mathbf{0}) \propto \delta^{ij}$ we find

$$\hat{p}^i \hat{p}^j Im^*\Pi^{ij}(\omega, \mathbf{0}) = \frac{1}{3} Im^*\Pi^{ii}(\omega, \mathbf{0}) = Im^*\Pi_t(\omega, \mathbf{0}) \,. \tag{14}$$

Finally, using $\omega_{l/t}(p = 0) = m_g$ and Eq. (13) with (14) in Eq. (12), we derive that

$$\gamma_l(m_g, 0) = \gamma_t(m_g, 0) \tag{15}$$

holds, as is expected on physical grounds.

Finally it is interesting to observe what consequences arise if we assume, on physical grounds, that $\gamma_l(m_g, 0) = \gamma_g(m_g, 0)$ to all orders in perturbation theory. From Eqs. (11), (12) this assumption leads to

$$\lim_{p \to 0} \left(\frac{\omega}{p}\right)^2 Im\Pi^{00} = \hat{p}^i \hat{p}^j Im\Pi^{ij}(\omega, \mathbf{0}),$$

and using this in Eq. (4) one finds

$$\lim_{p \to 0} \frac{1}{p^2} \Pi_g^\mu [\Pi_g^\nu + 2\delta^{\nu i} p^i] \, Im\Pi_{\mu\nu} = 0 \,.$$

## 4 Conclusions and remarks

The Slavnov-Taylor identity for gluon polarization tensor we report on in this letter, Eq. (1), allows to obtain various constrains. Applying it to the next to the leading order in HTL-resummed perturbation theory, the identity Eq. (7) results, which is relevant to leading-order damping rate of gluonic modes. As has been discussed, for zero three momentum the expected equality $\gamma_t(m_g, 0) = \gamma_l(m_g, 0)$ can be derived with the help of the identity Eq. (7). One of the advantages of the above procedure is that the explicit expansion of ${}^*\Pi^{00}(\omega_l, p)$ around $p \sim 0$, which has been found to be troublesome, can be avoided in this derivation. Moreover, concerning the direct calculation of $\gamma_l$, the present work indicates that dealing with singularities in intermediate steps of the calculation and necessary changes in the integration variables should be made in a way consistent with the identity Eq. (7).

**Appendix Derivation of (1) and (6)**

We start in the imaginary-time formalism continuing to real energies at the final stage. The QCD action in Coulomb gauge reads

$$
\begin{aligned}
S &= \int_0^{1/T} dx_0 \int d^3x \mathcal{L}(x) \quad \left( \equiv \int_T d^4x \, \mathcal{L}(x) \right) , \\
\mathcal{L}(x) &= -\frac{1}{4} F_a^{\mu\nu}(x) F_a^{\mu\nu}(x) - \frac{1}{2\lambda} (\nabla \cdot \mathbf{A}_a(x))^2 + \delta^{\mu i} \bar{\eta}_a(x) \partial^i D^\mu_{x,ab}(A) \eta_b(x) , \\
F_a^{\mu\nu} &= \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g C_{abc} A_b^\mu A_c^\nu .
\end{aligned} \tag{16}
$$

Here $D^\mu_{x,ab}(A) \equiv \delta_{ab} \partial/\partial x^\mu - g C_{abc} A_c^\mu(x)$ and $\bar{\eta}_a$ and $\eta_a$ are the Coulomb-ghost fields. For notations related to the color space, we follow [9].

The bare gluon propagator $\tilde{\Delta}^{\mu\nu}(P_E)$ reads

$$
\tilde{\Delta}^{\mu\nu}(P_E) = \delta^{\mu i} \delta^{\nu j} \frac{\delta^{ij} - \hat{p}^i \hat{p}^j}{P_E^2} + \delta^{\mu 0} \delta^{\nu 0} \frac{1}{p^2} + \lambda \frac{P_E^\mu P_E^\nu}{p^4} , \tag{17}
$$

where $P_E^\mu = (p_0, \mathbf{p})$ with $p_0 = 2\pi T n$ $(n = 0 \pm 1, \pm 2, ...)$, and the bare ghost propagator reads $\tilde{\Delta}_g(p) = 1/p^2$. The form of gluon-ghost vertex can be read off from $\mathcal{L} \ni g C_{abc} (\partial^j \bar{\eta}_a) \eta_b A_c^j$,

$$
\mathcal{V}_g = -i g C_{abc} \, p^j , \tag{18}
$$

where $\mathbf{p}$ is the outgoing momentum carried by $\bar{\eta}_a$.

The generating functional reads

$$
\begin{aligned}
Z[J, \bar{\xi}, \xi] = \int \mathcal{D}A_a^\mu \mathcal{D}\eta_a \mathcal{D}\bar{\eta}_a \exp \Bigg[ S &+ \int_T d^4x \, \{ J_a^\mu(x) A_a^\mu(x) \\
&+ \bar{\xi}_a(x) \eta_a(x) + \bar{\eta}_a(x) \xi_a(x) \} \Bigg] ,
\end{aligned} \tag{19}
$$

where, the functional integral is to be performed with periodic boundary conditions for all fields (including ghosts [10]), $A_a^\mu(x_0 = 0, \mathbf{x}) = A_a^\mu(x_0 = 1/T, \mathbf{x})$, etc.

The action $S$ (or $\mathcal{L}$), Eq. (16), is invariant under the BRST transformation [9]:

$$
\delta A_a^\mu = D_{ab}^\mu(A) \eta_b \, \delta\zeta , \quad \delta \bar{\eta}_a = -\frac{1}{\lambda} \nabla \cdot \mathbf{A}_a \, \delta\zeta , \quad \delta \eta_a = \frac{g}{2} C_{abc} \eta_b \eta_c \, \delta\zeta ,
$$

where $\delta\zeta$ is a Grassmann-number parameter. Using this fact in (19), we obtain,

$$\begin{aligned}
&\int\limits_T d^4z\mathcal{B}(z)Z[J,\bar{\xi},\xi]=0\,,\\
&\mathcal{B}(z)=J^\mu_a(z)D^\mu_{z,ab}\left(\frac{\delta}{\delta J}\right)\frac{\delta}{\bar{\xi}_b(z)}+\frac{1}{\lambda}\xi_a(z)\frac{\partial}{\partial z^i}\frac{\delta}{\delta J^i_a(z)}\\
&\qquad\quad+\frac{g}{2}C_{abc}\bar{\xi}_a(z)\frac{\delta}{\delta\bar{\xi}_b(z)}\frac{\delta}{\delta\bar{\xi}_c(z)}\,.
\end{aligned}\tag{20}$$

Computing

$$\frac{\delta}{\delta J^\mu_a(x)}\frac{\delta}{\delta\xi_b(y)}\int\limits_T d^4z\mathcal{B}(z)\ln Z[J,\bar{\xi},\xi]\Big|_{J\,=\,\bar{\xi}\,=\,\xi\,=\,0}\,,$$

by using (20), we obtain

$$\delta_{ab}\partial^\mu_x\tilde{G}_g(x-y)-gC_{acd}\langle A^\mu_d(x)\eta_c(x)\bar{\eta}_b(y)\rangle+\delta_{ab}\frac{1}{\lambda}\frac{\partial}{\partial y^i}\tilde{G}^{\mu i}(x-y)=0\,.\tag{21}$$

Note that the third term of Eq. (20) does not contribute to (21). The ghost propagator $\tilde{G}_g$ and the gluon propagator $\tilde{G}^{\mu\nu}$ in Eq. (21) are defined, respectively, through

$$\begin{aligned}
\delta_{ab}\tilde{G}_g(x-y)&=\langle\eta_a(x)\bar{\eta}_b(y)\rangle\equiv\frac{\delta\ln Z}{\delta\xi_b(y)\delta\bar{\xi}_a(x)}\Big|_{J\,=\,\bar{\xi}\,=\,\xi\,=\,0}\,,\\
\delta_{ab}\tilde{G}^{\mu\nu}(x-y)&=\langle A^\mu_a(x)A^\nu_b(y)\rangle\equiv\frac{\delta\ln Z}{\delta J^\mu_a(x)\delta J^\nu_b(y)}\Big|_{J\,=\,\bar{\xi}\,=\,\xi\,=\,0}\,.
\end{aligned}$$

$\langle A^\mu_d(x)\eta_c(x)\bar{\eta}_b(y)\rangle$ is the gluon-ghost three-point function.

Now, we call for the Schwinger-Dyson equations:

$$\begin{aligned}
\tilde{G}^{\mu\nu}(x-y)&=\tilde{\Delta}^{\mu\nu}(x-y)\\
&\quad-\int\limits_T d^4ud^4v\,\tilde{G}^{\mu\rho}(x-u)\tilde{\Pi}^{\rho\sigma}(u-v)\tilde{\Delta}^{\sigma\nu}(v-y)\,,
\end{aligned}\tag{22}$$

$$\begin{aligned}
\delta_{ab}\tilde{G}_g(x-y)&=\delta_{ab}\tilde{\Delta}_g(x-y)\\
&\quad+gC_{ade}\int\limits_T d^4z\frac{\partial\tilde{\Delta}_g(x-z)}{\partial z^i}\langle A^i_e(z)\eta_d(z)\bar{\eta}_b(y)\rangle\,,
\end{aligned}\tag{23}$$

where $\tilde{\Pi}^{\rho\sigma}$ is the gluon polarization tensor. It can readily be shown that, in the limit $g \to 0$, (21) holds, as it should be. Taking this fact into account, we substitute (22) and (23) into (21) and use the form (17) for $\tilde{\Delta}^{\sigma i}$ to obtain

$$\begin{aligned} \delta_{ab} \int_T d^4z \tilde{\Pi}^{\nu\mu}(x-z) \partial_y^\mu \hat{\Delta}_g(z-y) \\ = -g C_{acd} \int_T d^4\xi \int_T d^4z (\tilde{G}^{-1})^{\nu\mu}(x-\xi) \\ \times \left[ \delta^{\rho i} \partial_\xi^\mu \partial_\xi^i \tilde{\Delta}_g(\xi - z) + \delta^4(\xi - z) \delta^{\mu\rho} \right] \langle A_d^\rho(z) \eta_c(z) \bar{\eta}_b(y) \rangle \, . \end{aligned} \tag{24}$$

The inverse propagator $(\hat{G}^{-1})^{\nu\mu}$ is related to $\tilde{\Pi}^{\nu\mu}$ through $(\tilde{G}^{-1})^{\nu\mu} = (\tilde{\Delta}^{-1})^{\nu\mu} + \tilde{\Pi}^{\nu\mu}$. The Fourier transform of $\langle A_d^\rho(z) \eta_c(z) \bar{\eta}_b(y) \rangle$ may be written as

$$g C_{acd} \left[ \langle A_d^\rho(z) \eta_c(z) \bar{\eta}_b(y) \rangle \right]_{\mathrm{F.T.}} = -i \delta_{ab} \tilde{\Pi}_g^\rho(P_E) \tilde{G}_g(P_E) \, , \tag{25}$$

where $(\tilde{\Pi}_g^\rho(P_E))_{ab}$ is related to a ghost self-energy part $\tilde{\Pi}_g$ through

$$p^i \tilde{\Pi}_g^i(P_E) = \tilde{\Pi}_g(P_E) \, . \tag{26}$$

Putting altogether in (24) and transforming to Fourier space, we finally obtain

$$\begin{aligned} \tilde{\Pi}^{\nu\mu}(P_E) P_E^\mu = -p^2 \tilde{G}_g(P_E) \left[ \left\{ \delta^{\nu\mu} P_E^2 - P_E^\nu P_E^\mu + \tilde{\Pi}^{\nu\mu}(P_E) \right\} \tilde{\Pi}_g^\mu(P_E) \right. \\ \left. - \frac{P_E^\mu}{p^2} \tilde{\Pi}^{\mu\nu}(P_E) \tilde{\Pi}_g(P_E) \right] \, , \end{aligned} \tag{27}$$

where use has been made of (26). Noting that $\tilde{G}_g(P_E) = [p^2 + \tilde{\Pi}_g(P_E)]^{-1}$ and using $\tilde{\Pi}^{\mu\nu} = \tilde{\Pi}^{\nu\mu}$, Eq. (27) may be 'solved' as

$$P_E^\nu \tilde{\Pi}^{\mu\nu}(P_E) = - \left[ \delta^{\mu\nu} P_E^2 - P_E^\mu P_E^\nu + \tilde{\Pi}^{\mu\nu}(P_E) \right] \tilde{\Pi}_g^\nu(P_E) \, . \tag{28}$$

¿From this we obtain Eq. (1) in the main text after continuation to real energies $ip_0 \to \omega + i0^+$ according to

$$\begin{aligned} & P^\mu = (\omega, \mathbf{p}) \, , \quad P_E^2 \to -P^2 = p^2 - \omega^2 \, , \\ & \tilde{\Pi}^{\mu\nu}(ip_0, \mathbf{p}) \to (-i)^{\delta_{\mu 0} + \delta_{\nu 0}} \Pi^{\mu\nu}(\omega, \mathbf{p}) \, , \quad \tilde{\Pi}_g^\mu(ip_0, \mathbf{p}) \to (-i)^{\delta_{\mu 0}} \Pi_g^\mu(\omega, \mathbf{p}) \, . \end{aligned} \tag{29}$$

Finally we compute $\Pi_g^\mu(P)$ with soft $P$ to lowest non-trivial order — one-loop

contribution — as required for the discussion in the main text. Using Eqs. (17) and (18), we have

$$\begin{aligned}\tilde{\Pi}^{\mu}_{g}(P_E) &= -g^2 N\, T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} \frac{(p^i - k^i)}{(\mathbf{p}-\mathbf{k})^2} \tilde{\Delta}^{i\mu}(K_E) \\ &= -g^2 N\, T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} \frac{p^i - k^i}{(\mathbf{p}-\mathbf{k})^2} \left[ \delta^{\mu j} \delta^{ij}_{\perp}(\hat{\mathbf{k}}) \tilde{\Delta}(K_E) + \lambda \frac{k^i K^{\mu}_E}{k^4} \right], \end{aligned} \tag{30}$$

where $p^i = (0, \mathbf{p})$, $K^{\mu}_E = (k_0, \mathbf{k})$ and $\delta_{\perp}{}^{ij}(\hat{\mathbf{k}})$ is as in (11). In Eq. (30), $\tilde{\Delta}(K_E) = 1/K_E^2$ for hard $p$, while, for soft $p$, the HTL-resummed ${}^*\tilde{\Delta}(K_E)$ is substituted for $\tilde{\Delta}(K_E)$. After carrying out the renormalization, one can easily see that the leading contribution comes from the first term (in the square brackets) with soft $k$ region:

$$\begin{aligned}\tilde{\Pi}^{\mu}_{g}(P_E) \simeq{}& -g^2 N\, T \sum_{k_0} \int_{\text{soft } k} \frac{d^3k}{(2\pi)^3} \frac{(p^i - k^i)}{(\mathbf{p}-\mathbf{k})^2} \delta^{\mu j} \delta^{ij}_{\perp}(\hat{\mathbf{k}}) \\ & \times \int_0^{1/T} d\tau e^{ik_0\tau} \int d\zeta\, \rho_t(\zeta, k)[1 + n(\zeta)] e^{-\zeta\tau}, \end{aligned} \tag{31}$$

where $\rho_t(\zeta, k)$ is the spectral function of the HTL-resummed transverse gluon propagator and $n(\zeta) = 1/(e^{\zeta/T} - 1)$. On summing over $k_0$ and using $n(\zeta) \simeq T/\zeta$ and $\int d\zeta \rho_t(\zeta, k)/\zeta = 1/k^2$, we obtain

$$\begin{aligned}\tilde{\Pi}^{\mu}_{g}(P_E) &\simeq -g^2 NT \delta^{\mu i} \int_{\text{soft } k} \frac{d^3k}{(2\pi)^3} \frac{p^i - (\mathbf{p}\cdot\hat{\mathbf{k}})\hat{k}^i}{k^2(\mathbf{p}-\mathbf{k})^2} \\ &\simeq -\delta^{\mu i} \frac{g^2 NT}{16} \hat{p}^i, \end{aligned} \tag{32}$$

which is independent of $\lambda$. Noting that $\tilde{\Pi}^{\mu}_{g}$ in (32) has only spatial components and is independent of $p_0$, we have $\Pi^{\mu}_{g}(P) = \tilde{\Pi}^{\mu}_{g}(P_E)$ (cf. Eq. (29)).


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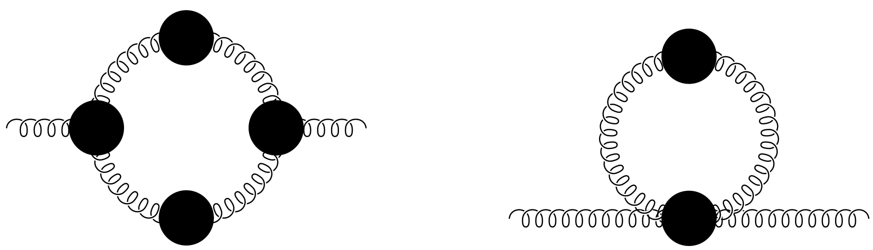

Fig. 1. Next to leading order contributions to the imaginary part of the gluon polarization tensor. All loop-momenta are soft and HTL-resummed propagators and vertices have to be used.